A Data-driven Approach For Drug Replenishment In Hospital Wards

Tommaso Levato, Leonardo Galli, Federica Picca Nicolino, Fabio Schoen, Luca Tigli, Filippo Visintin

University of Florence

LINFA

Logistica INtelligente del Farmaco

LINFA Project
(Intelligent Logistics of Pharmaceuticals)

What:

Better policies for drug replenishment

How:

Data driven methods

Introduction

Goal: One order per week

Framework:

- Inventory management
- Demand forecasting

Inventory management

Models

- Lot-Sizing Model

$$\begin{aligned}
\min_{x,m,\delta} \sum_{t=1}^{7} & C_t(x_t) + H_t(m_t) + K_t\delta_t \\
\text{s.t. } & m_t + x_t = d_t + m_{t+1} && \forall\, t = 1, \ldots, 7 \\
& x_t \leq \delta_t M && \forall\, t = 1, \ldots, 7 \\
& x_t, m_t \in \mathbb{Z}^+ && \forall\, t = 1, \ldots, 7 \\
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Demand forecasting

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Dataset: $\{(x_i, y_i), i = 1, \ldots, N\}$

- Synthetic data
- 20 years of history

Features:

- number of patients in ward
- average length of stay
- quartiles of length of stay

Labels:

- weekly demands: $[1, 4, 2, 3, 1, 2, 2]$

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Machine learning models

- KNN
- Decision Trees
- Random Forests

Data Driven Optimization

Approaches

- Predictive approach:

$$\min_{z} c(z; \hat{y})$$

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Approaches

- Stochastic approach:

$$\min_{z} \frac{1}{N} \sum_{i=1}^{N} c(z; y_i)$$

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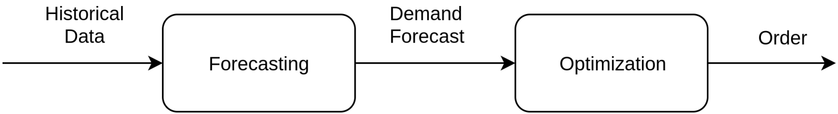
Evaluation

Monday:



If a **stockout** occurs:

- Repeat the process!

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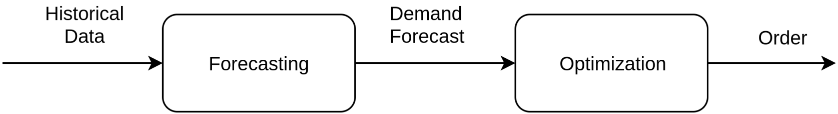

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Evaluation

At the end of the week:

- $\text{Cost} = \sum_{t=1}^{7} C_t(\bar{x}_t) + H_t(\bar{m}_t) + K_t\bar{\delta}_t$

where

- $\bar{\boldsymbol{x}}$: actual order
- $\bar{\boldsymbol{m}}$: actual inventory
- $\bar{\boldsymbol{\delta}}$: actual order activation

Results

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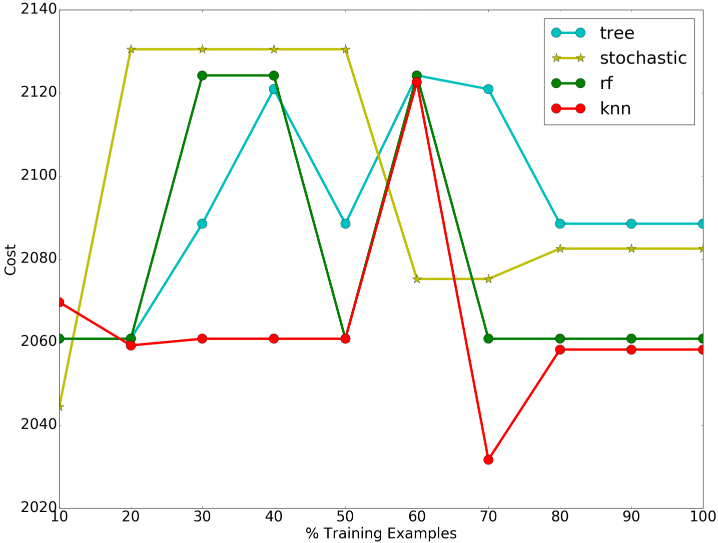
Model	Cost
Stochastic	2082.5
KNN	**2058.2**
Tree	2088.5
Random Forest	2060.8

Results


tree
stochastic
rf
knn
Cost
% Training Examples
2140
2120
2100
2080
2060
2040
2020
10
20
30
40
50
60
70
80
90
100

Questions?